

This article was downloaded by:

On: 22 January 2011

Access details: *Access Details: Free Access*

Publisher *Taylor & Francis*

Informa Ltd Registered in England and Wales Registered Number: 1072954 Registered office: Mortimer House, 37-41 Mortimer Street, London W1T 3JH, UK



## **The Journal of Adhesion**

Publication details, including instructions for authors and subscription information:

<http://www.informaworld.com/smpp/title~content=t713453635>

## **The Effect of Adhesive Thickness on Joint Strength**

W. A. Dukes<sup>a</sup>; R. W. Bryant<sup>a</sup>

<sup>a</sup> Ministry of Technology Waltham Abbey, Essex, England

**To cite this Article** Dukes, W. A. and Bryant, R. W.(1969) 'The Effect of Adhesive Thickness on Joint Strength', The Journal of Adhesion, 1: 1, 48 – 53

**To link to this Article:** DOI: 10.1080/00218466908077374

**URL:** <http://dx.doi.org/10.1080/00218466908077374>

PLEASE SCROLL DOWN FOR ARTICLE

Full terms and conditions of use: <http://www.informaworld.com/terms-and-conditions-of-access.pdf>

This article may be used for research, teaching and private study purposes. Any substantial or systematic reproduction, re-distribution, re-selling, loan or sub-licensing, systematic supply or distribution in any form to anyone is expressly forbidden.

The publisher does not give any warranty express or implied or make any representation that the contents will be complete or accurate or up to date. The accuracy of any instructions, formulae and drug doses should be independently verified with primary sources. The publisher shall not be liable for any loss, actions, claims, proceedings, demand or costs or damages whatsoever or howsoever caused arising directly or indirectly in connection with or arising out of the use of this material.

# The Effect of Adhesive Thickness on Joint Strength

W. A. DUKES AND R. W. BRYANT

*Ministry of Technology  
Waltham Abbey, Essex, England*

(Received November 12, 1968)

## ABSTRACT

The hypothesis that the weakness of thick joints is due to stresses caused by contraction of the adhesive on setting requires a linear relation between the strength of a joint and the adhesive thickness. However it is statistically preferable to describe our experimental data by relating either log strength or linear strength to log thickness. The predicted slopes or thickness-dependences of the joint strengths are at least three orders greater than those observed. Both the preferred relations have a possible theoretical basis, namely the statistical nature of rupture.

## INTRODUCTION

WAKE [1, 2] HAS developed an argument that the weakness of thick joints when compared with thin ones is due to stresses built into the joint by the contraction of the adhesive on setting, or by differential contraction between adhesive and adherend after curing at an elevated temperature. His treatment may be summarised as follows. In a 'poker-chip' butt joint, to be tested in tension, contraction of the disc of adhesive, of original thickness  $d_o$ , is first notionally allowed to take place, and then a radial stress is applied to counteract it, causing a strained thickness  $d_o/r^2$  where  $r$  is the ratio of the strained to the unstrained radii. For any material subjected to a finite deformation, large compared with that normally considered in classical elasticity theory, the radial stress is given by  $G(r^2 - 1/r^4)$ , where  $G$  is the shear modulus, which expression when multiplied by the strained thickness gives "the total force acting across a section of unit length . . . acting as if it were . . . in the plane of the two interfaces" [1], equal to  $G d_o(1 - 1/r^6)$ . On breaking in axial tension, the total work done  $W$  is taken as equal to the energy to rupture the two adhesive bonds plus the work done by the adhesive disc when freed from constraints [2]:

$$W = 2fl + 2Gd_o r_o \pi (1 - 1/r^6) \quad (1)$$

where  $f$  = force to rupture bonds,  $l$  = distance over which adhesive forces act and  $r_o$  = radial contraction of disc

*The Effect of Adhesive Thickness on Joint Strength*

$$\text{i.e. } f = \frac{W}{2l} - \frac{Gd_o r_o \pi}{l} (1 - 1/r^6) \quad (2)$$

Thus the theoretical force to rupture a bond is subject to a negative correction increasing directly with increasing thickness of adhesive layer [2].

Taking  $R_u$  as the unstrained and  $R_s$  as the strained radii, then by definition

$$R_u \cdot r = R_s \quad (3)$$

Taking  $\alpha$  as the linear contraction (inch/inch) on setting at room temperature, then

$$R_s - R_s \cdot \alpha = R_u \quad (4)$$

Thus from Eqn. 3

$$1 - \alpha = 1/r \quad (5)$$

The radial contraction

$$r_o = R_s - R_u = R_s \alpha = \frac{R_s(r - 1)}{r} \quad (6)$$

So  $r_o$  may be eliminated from Eqn. 2 to give:

$$f = \frac{W}{2l} - \frac{\pi R_s G d_o}{l} [1 + 1/r^7 - (1/r^6 + 1/r)] \quad (7)$$

Thus the negative correction to the theoretical force to break the bond is predicted to be proportional to (a) the specimen's radius, (b) the adhesive's shear modulus, (c) the adhesive thickness and (d) the  $r$  terms within the brackets, which together increase steeply as a function of  $\alpha$ . The value of  $l$  is not known, but has been assumed [3] to be in the order of  $1 \times 10^{-8}$  in.

Alternatively, the argument leading to Eqns. 1, 2 and 7 can be modified as follows. The radial stress should be multiplied by the cylindrical area (equal to the strained thickness times the strained perimeter  $2\pi R_s$ ), to give the total force  $Gd_o(1 - 1/r^6) \cdot 2\pi R_s$ . Then the total work done on breaking in axial tension (cf. Eqn. 1) would be approximated by the energy to rupture the two adhesive bonds plus the work done by the adhesive disc, taken to be the product of the force and half the radial contraction (since the force diminishes to zero through the total contraction):

$$W = 2fl + Gd_o r_o \pi (1 - 1/r^6) R_s \quad (8)$$

Again eliminating  $r_o$ , the force to rupture the bonds will be

$$f = \frac{W}{2l} - \frac{\pi R_s^2 G d_o}{2l} [1 + 1/r^7 - (1/r^6 + 1/r)] \quad (9)$$

In contrast with Eqn. 7, this predicts that the negative correction to the theoretical force to break the bond is proportional to the square of the specimen's radius, and not to the radius itself.

## COMPARISON OF THEORETICAL AND EXPERIMENTAL RELATIONSHIPS

Bryant and Dukes [4, 5, 6] have made many measurements of the strengths of joints over a wide range of thicknesses, using four different joint designs and two room-temperature curing adhesives, one a rubber and the other a tough epoxy, not very dissimilar to polystyrene, previously considered [2] in this context. Although many of their measurements have been in shear, rather than tension, it is thought worthwhile to compare their results in both modes with Eqns. 7 and 9, as the above energy consideration should be applicable to either mode of testing. The measurements were made over a wide range of strain-rates at each adhesive thickness, and for the present purpose the strain-rate parameter has been eliminated by interpolating to a standard true rate of  $1 \text{ sec}^{-1}$  (corrected for deformation in machine and linkages) or alternatively to a time-to-break of 1 min. For any given joint design and adhesive thickness the highest correlations were found between joint strength and strain-rate or time-to-break with both on logarithmic scales, and the appropriate least-squares regression was used to normalise each strength determination to the standard rate (or time-to-break). The use of the log/log relationship rather than another does not affect what follows, since the particular relationship chosen would make little difference to an interpolation (but not of course to an extrapolation).

There resulted for each joint design a number of normalised strength determinations at each adhesive thickness. Three different relations between normalised strength and thickness have been examined statistically—(a) linear strength against linear thickness (i.e., 'plain/plain'), (b) log strength against log thickness (i.e., 'log/log'), and (c) linear strength against log thickness (i.e., 'plain/log'). The statistical parameters of each relation for each combination of adhesive and joint design are shown in Table 1, together with values of shear modulus and linear contraction as measured for each adhesive.

The linear (plain/plain) relation in every case has a lower correlation coefficient than either of the other relations. Comparison of the Fisher  $z$ -transformations is made in Table 2, in which the degree of significance is derived from tables of Student's  $t$ . Using MS.9160 adhesive, the plain/plain correlation coefficients are significantly lower than those for the log/log relation in four out of five comparisons (the fifth does not quite reach the 5% level), but with AY.103 adhesive the plain/plain coefficients are not significantly lower when compared with those of either of the other relations. The least scattered set (MS.9160/Al) shows the most adverse comparisons against the plain/plain relation. In six out of seven comparisons the log/log coefficients are higher than the plain/log ones, but only one of these is statistically significant. In the only case in which a plain/log coefficient is greater than the log/log one, the difference is insignificant.

Table 1. Statistical Parameters of Different Relations Between Joint Strength and Adhesive Thickness

Adhesive	Shear Modulus G, p.s.i.	Linear Contraction from gelation to test condition, $\alpha_i$ in/in	From Eqn. 5 $\frac{[1 + 1/r^7] - (1/r^6)}{1/r}$	Substrate	Joint Design $\phi$	Joint No. of Tests	Thickness Range $\times 10^{-3}$	Negative Statistical Parameters† of normalised strength/thickness relations plotted:											
								Linearly (plain/plain)				Log/Log				Plain/Log			
								m	r	z	$\phi$	m	r	z	$\phi$	m	r	z	$\phi$
Silicone Rubber MS.9160	$1.5 \times 10^2$	0.0077	0.00035	Steel	●	45	5.3-46.6	2,352	0.824	1.168	0.208	0.968	2.053	113.9	0.873	1.345			
					◇	77	4-47	3,768	0.814	1.139	0.283	0.897	1.453	171.4	0.891	1.426			
					○	18	4-20	Scatters too large to distinguish thickness											
					◇	45	3.75-47.7	4,985	0.924	1.616	0.351	0.981	2.329	208.6	0.972	2.135			
					◇	37	3.7-46.2	5,249	0.935	1.646	0.380	0.985	2.450	239.4	0.987	2.515			
Steel	◇	55	3.6-92.1	1,045	0.798	1.092	0.132	0.905	1.497	79.4	0.902	1.483							
Tough Epoxy AY.103	$1.5 \times 10^5$	0.0044	0.00010	Steel	●	45	4-20	Scatters too large to distinguish thickness											
					◇	68	0.1-20	55,200	0.578	0.659	0.103	0.714	0.895	707.6	0.710	0.886			
					○	28	4-20	Scatters too large to distinguish thickness											
					◇	99	0.33-20.1	22,630	0.294	0.303	0.049	0.341	0.356	424.3	0.323	0.335			
Steel	◇	99	0.33-20.1	22,630	0.294	0.303	0.049	0.341	0.356	424.3	0.323	0.335							

$\phi$  Test Mode  
 Shear  
 Tension  
 Joint Design  
 Circular Butts in Torsion ●  
 Circular Butts in Simple Shear ◇  
 Tubular Butts in Torsion ○  
 Circular Butts in Tension (poker-chips) ◀||→  
 †m = slope  
 r = correlation coefficient  
 z = Fisher's transformation

**Table 2. Statistical Comparison of Different Relations Between Joint Strength and Adhesive Thickness**

Adhesive	Substrate	Joint Design $\phi$	Standard Error of $\Delta z$	Adverse Comparisons against each Relation in turn, measured by $\Delta z/S.E.$ :					
				Vs. Linear (Plain/Plain)		Vs. Log/Log		Vs. Plain/Log	
				Log/Log	Plain/Log	Plain/Plain	Plain/Log	Plain/Plain	Log/Log
MS.9160	Steel	●	0.218	4.1***	(0.8)	—	—	—	3.2**
	Steel	◇	0.164	(1.9)	(1.8)	—	—	—	(0.2)
	Steel	◇	0.218	3.3**	2.4*	—	—	—	(0.9)
	Al	◇	0.243	3.3**	3.6***	—	(0.3)	—	—
	Steel	←  →	0.196	2.1*	(2.0)	—	—	—	(0.7)
AY.103	Steel	◇	0.175	(1.4)	(1.3)	—	—	—	(0.1)
	Steel	←  →	0.144	(0.4)	(0.2)	—	—	—	(0.1)

\* Significant at the 5% level  
 \*\* Significant at the 1% level  
 \*\*\* Significant at the 0.1% level

Thus statistically it seems preferable to describe these measurements by relating either log strength or linear strength to log thickness rather than linear strength to linear thickness, as required by Eqns. 7 and 9.

**QUANTITATIVE COMPARISON OF THEORY AND EXPERIMENT**

However it is of interest to compare the measured plain/plain data with the theory. The theoretical strength correction factor,

$$\pi R_s G [1 + 1/r^7 - (1/r^6 + 1/r)] / l$$

has been calculated for each adhesive, by substituting in Eqns. 5 and 7 the measured values of  $G$  and  $\alpha$ , taking  $\pi R_s = 2$  in. and  $l = 1 \times 10^{-8}$  in.; it is in the order of  $10^7$  psi/in. for MS.9160 and  $3 \times 10^9$  psi/in. for AY.103. (If Eqn. 9 rather than Eqn. 7 were used, these figures would be reduced by a factor of about 1/3). This theoretical figure (from Eqn. 7) for MS.9160 is  $2 \times 10^3$  to  $4 \times 10^3$  times greater than the measured linear (plain/plain) correction factors in shear, and  $10^4$  times greater in tension. For AY.103 it is  $5 \times 10^4$  and  $10^5$  times greater respectively. It is probably a coincidence that each theoretical figure is roughly the square of the value measured in shear. Wake [2] chose an arbitrary value for  $l$  of  $5 \times 10^{-4}$  (cf.  $10^{-8}$  in.) in order to fit the data for polystyrene (with a modulus close to that of AY.103), and this admittedly would reduce the theoretical figure for MS.9160 to about one-fifteenth of the value measured in shear and one-fifth of that in tension, and

## *The Effect of Adhesive Thickness on Joint Strength*

would remove the discrepancy for AY.103 in shear, leaving the theoretical figure less than three times the value measured for AY.103 in tension.

When the two adhesives are compared, it is seen that a reduction of linear contraction by a factor of 4/7 together with a thousand-fold increase in  $G$  is associated with merely a twenty-fold increase in the linear correction factor, measured in shear or tension, and not 570-fold as predicted by (7).

### CONCLUSION

The hypothesis that the weakness of thick joints is due to stresses caused by contraction of the adhesive on setting requires a linear relation between the strength of a joint and the adhesive thickness. It is however statistically preferable to describe our experimental data by relating either log strength or linear strength to log thickness. The predicted slopes or thickness-dependences of the joint strengths are at least three orders greater than those observed.

The hypothesis assumes adhesive failure at the interface, which was not observed in any of our experiments. Cohesive failure might well favour the following argument. Both the preferred relations (log/log and plain/log) between joint strength and adhesive thickness have a possible theoretical basis [7], in the analysis by extreme value statistics of the negatively skewed frequency distribution of tensile strengths (of rubbers), involving the frequency function of the maximum flaw. The former relation would hold if a Weibull type of distribution is used, and the latter if an exponential one. Thus it is possible that the dependence of joint strength on adhesive thickness is a consequence of the statistical basis of rupture.

### REFERENCES

1. W. C. Wake, *Trans. I.R.I.* 35, (5) (1959), p. 146.
2. W. C. Wake, in *Adhesion*, ed. D. D. Eley, Oxford, (1961), p. 202.
3. D. D. Eley and D. Tabor, *ibid.*, p. 9.
4. R. W. Bryant and W. A. Dukes, *Applied Polymer Symposia*, 3, (1966), p. 81. Interscience.
5. *Idem*, paper to Adhesion Conference, Nottingham (1966). To be published by MacLaren and Sons, London, in 'Adhesion, Fundamentals and Practice'.
6. *Idem*, paper to S.A.E., Los Angeles Oct. 1967, reference No. 670855, S.A.E., New York.
7. R. F. Landel and R. F. Fedors, in 'Fracture Processes in Polymeric Solids', ed. B. Rosen, Wiley (1964), p. 446ff.

British Crown Copyright, reproduced with the permission of the Controller, Her Britannic Majesty's Stationery Office.